OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4754/01A

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

FRIDAY 22 JUNE 2018: Morning

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4754/01A sent with the standard paper
MEI Examination Formulae and Tables (MF2) sent with the standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KS/TP) 172280/1

8025761263

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

This paper will be followed by PAPER B: COMPREHENSION.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Express $\sin\theta - 2.4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the maximum value of the function $f(\theta) = 1 - \sin\theta + 2.4\cos\theta$, where $0 \leqslant \theta \leqslant 2\pi$. [5]

2 The finite region bounded by the curve $y = \ln x$, the x-axis, the y-axis and the line $y = 1$ is rotated through 360° about the y-axis. Find the exact volume of the solid of revolution generated. [4]

3 Find the first three terms of the binomial expansion of $\dfrac{1+2x}{(2-x)^3}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. [7]

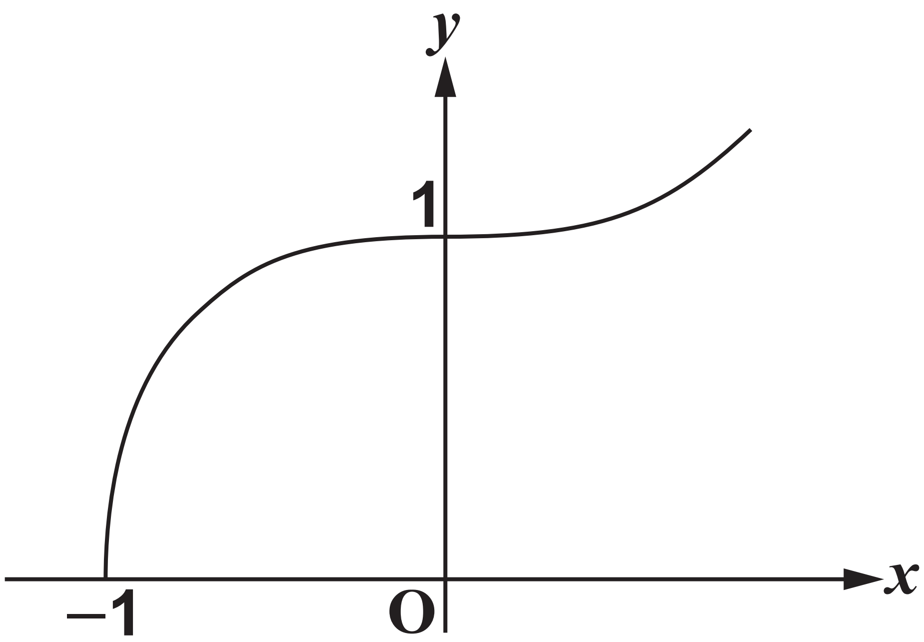
4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2\cos\theta - \cos 2\theta$, where $0 < \theta < \pi$.

(i) Find $\dfrac{dy}{dx}$ in terms of θ. [3]

(ii) Find the exact coordinates of the point on the curve where the gradient is zero. [4]

5 Fig. 5 shows the curve with equation $y = \sqrt{1 + x^3}$.

FIG. 5



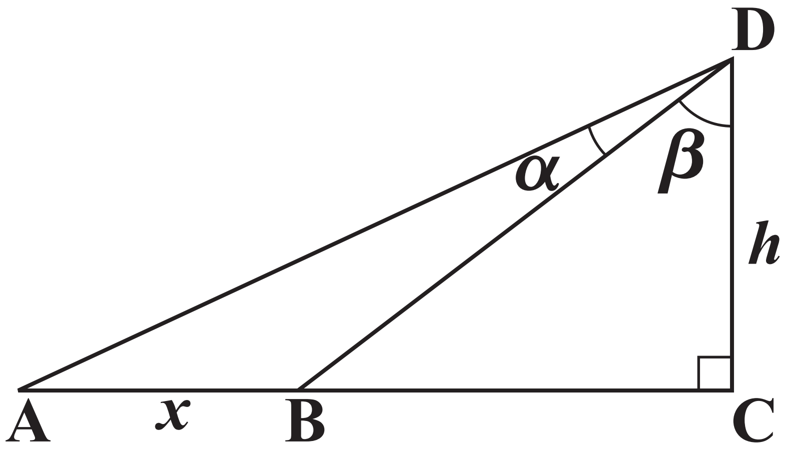
(i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the x- and y-axes, giving your answer correct to 3 significant figures. [3]

(ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. [1]

(iii) State, with a reason, which of these estimates is closer to the true area. [1]

6 **In Fig. 6, triangle ADC is right-angled at C, with CD = h. The point B on AC is such that AB = x, angle ADB = α and angle $BDC = \beta$.**

FIG. 6



(i) Find BC and AC in terms of h, α and β.

Hence show that $x = \dfrac{h \tan\alpha \sec^2\beta}{1 - \tan\alpha \tan\beta}$. [5]

(ii) Given that $x = h$ and $\beta = 30°$, find α, giving your answer correct to 1 decimal place. [3]

SECTION B (36 marks)

7 Three points A, B and C have coordinates A (2, 1, 1), B (1, −3, −1) and C (−4, −1, 0).

(i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC. [7]

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}),$$
$$\mathbf{r} = \mathbf{i} - 3\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$$
and $$\mathbf{r} = -4\mathbf{i} - \mathbf{j} + \nu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

pass through the points A, B and C respectively.

(ii) Show that these three lines meet at a point D. [6]

You are given that the plane ABC has equation $\mathbf{r}.(\mathbf{j} - 2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

(iii) Find the coordinates of E. [3]

The volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ height.

(iv) Find the volume of the tetrahedron ABCD. [3]

8 The speed $v\,\mathrm{m\,s^{-1}}$ of an object at time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -kv(4 + v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A, B and C such that

$$\frac{1}{v(4+v^2)} = \frac{A}{v} + \frac{Bv + C}{4 + v^2}.$$ [5]

(ii) Hence show by integration that $v = \dfrac{4}{\sqrt{5\mathrm{e}^{8kt} - 4}}$. [9]

(iii) After 1 second the speed of the object is $2\,\mathrm{m\,s^{-1}}$. Find the value of k. [3]

END OF QUESTION PAPER